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MATH 121B　　Calculus and Analytic Geometry I　　Fall 2004　　Final Exam

Part A Instructions: Do the work for these problems on separate paper. Clearly box or circle the final answer for each computational problem. Do reasonable simplification. Each problem has a value of 5 points. You may not use the symbolic features of a calculator such as those on a TI-89 for this part of the exam.

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1. State the definition of *derivative*.

2. State the definition of *definite integral*.

3. State the definition of *antiderivative*.

4. State the First Fundamental Theorem of Calculus. Include the hypotheses and the conclusion.

5. Evaluate the limit $\lim_{x \to 0} \dfrac{\sin x}{x}$.

6. Evaluate the limit $\lim_{x \to 5} \dfrac{x - 5}{x^2 - 25}$.

7. Evaluate the limit $\lim_{x \to \infty} \dfrac{x^3}{e^x}$.

8. Compute the derivative of $f(x) = 6x^3 - 12x^2 + 5x$.

9. Compute the derivative of $f(x) = x + \sin(x)$.

10. Compute the derivative of $g(t) = t\cos(t)$.

11. Compute the derivative of $h(x) = \sin(5x)$.

12. Compute the derivative of $f(x) = \dfrac{x^2}{x+1}$.

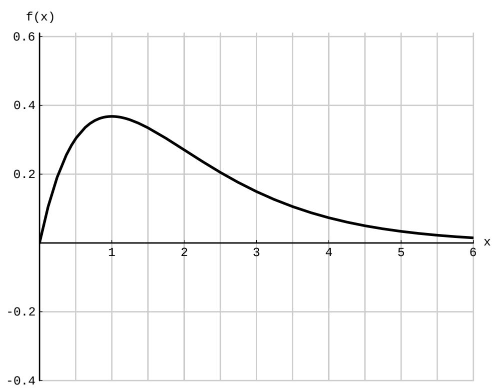
13. Compute the derivative of $g(t) = \ln(\sin(t))$.

14. Compute the derivative of $f(x) = \sqrt{1 + x^2}$.

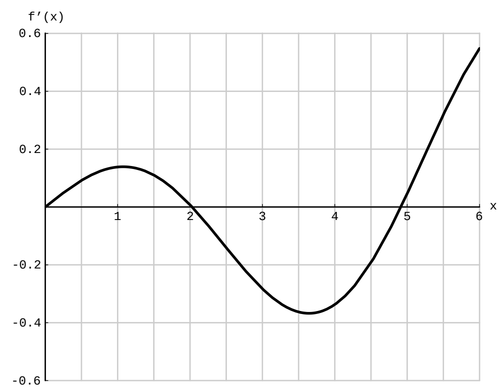
15. Compute the derivative of $f(x) = e^{3x} \sin(7x)$.

16. The plots on the left below show the graphs of three functions. The plots on the right show the graphs of the derivative functions for the three functions. Match each function with its derivative.

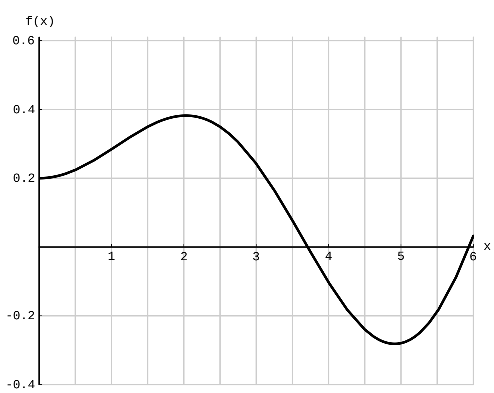
(a)



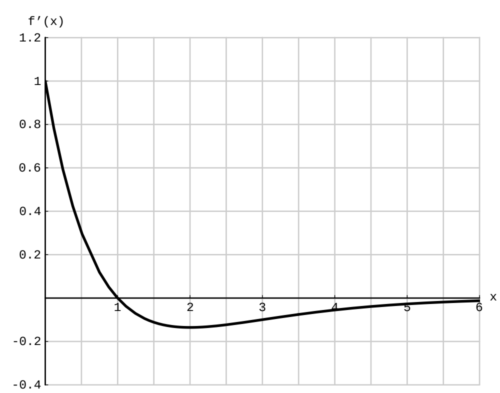
(i)



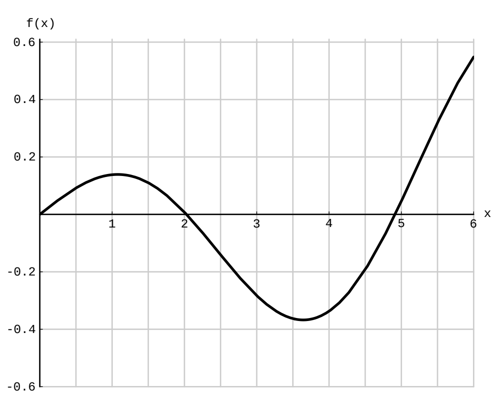
(b)



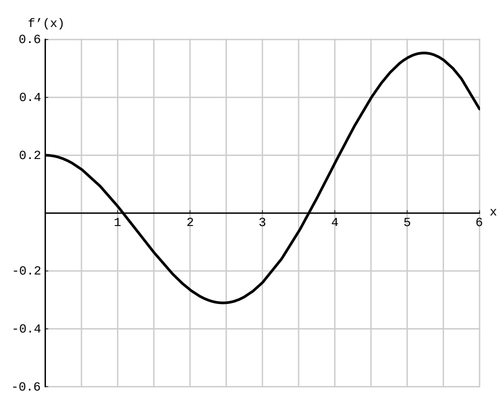
(ii)



(c)



(iii)



17. Compute the *second* derivative of $g(x) = 7x^3 + 4x^2$.

18. Find the slope of the graph of $f(x) = 5x^2 - 3x$ at $x = 1$.

19. Find the equation of the line tangent to the graph of $f(x) = x^3 + 4x$ at $x = 2$.

20. The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$. Find a formula for the rate of change in V with respect to r.

21. Find an antiderivative for $f(x) = x + \dfrac{1}{x}$.

22. Evaluate the indefinite integral $\displaystyle\int (x^2 - \sin x)\,dx$.

23. Evaluate the definite integral $\displaystyle\int_{-1}^{2} x^3\,dx$.

24. Evaluate the definite integral $\displaystyle\int_{0}^{1} (x + \sqrt{x})\,dx$.

25. The rate at which water flows into a pond is given by $f(t) = t^4$ in units of gallons per hour. Find the amount of water that accumulates between $t = 1$ and $t = 2$.

Part B Instructions: Do any three of the following five problems. Do the work for these problems on separate paper. On this page, circle each of the problem numbers for the three problems you are submitting. Each problem is worth 15 points.

I. Find the slope $\frac{dy}{dx}$ of the line tangent to the curve with equation $x^2y - xy^3 = 12$ at the point $(x, y) = (4, 1)$.

II. An inverted cone (i.e., pointy end down) is being filled with water at a rate of 2 cubic inches per minute. The cone is 15 inches high and has a radius of 5 inches. Find the rate at which the depth of the water is changing when the depth is 4 inches. Hint: The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

III. Consider the function $f(x) = x^3 - Ax^2$ where A is a positive constant. Use calculus techniques to do the following.

(a) Show that the function has a local minimum at $x = \frac{2}{3}A$.

(b) Show that the function has an inflection point at $x = \frac{1}{3}A$.

IV. You need to build a fence that encloses a rectangular region with an area of 100 square feet. One edge will be built with more expensive material because it faces the street. The cost for that edge is five times greater per foot than the cost for the other three edges. Find the dimensions of the rectangle that can be enclosed for minimum cost.

V. Approximate the value of $\displaystyle\int_0^1 e^{-x^2}\,dx$ using 10 rectangles and right endpoints.